MAT 273
CALCULUS III

COURSE DESCRIPTION:

Prerequisites: MAT 272
Corequisites: None

This course covers the calculus of several variables and is the third calculus course in a three-course sequence. Topics include functions of several variables, partial derivatives, multiple integrals, solid analytical geometry, vector-valued functions, and line and surface integrals. Upon completion, students should be able to solve problems involving vectors and functions of several variables. This course has been approved to satisfy the Comprehensive Articulation Agreement for the general education core requirement in natural sciences/mathematics. Course Hours Per Week: Class, 3. Lab, 2. Semester Hours Credit, 4.

LEARNING OUTCOMES:

1. Students will be able to convert between coordinate systems in three dimensions, recognize and graph standard equation forms in three dimensions, and carry out vector operations to solve practical problems. Students will display proficiency by demonstrating the following competencies:

a. Plot points in rectangular coordinates in space; find the distance between two points in space.
b. State the definition of and give the standard form of the equation of a sphere; identify the center and radius.
c. Find a vector with given initial and terminal points and sketch its position-vector representation; find the magnitude and direction of a vector; find sums, differences, and scalar products of vectors; express vectors in terms of the perpendicular unit vectors ***i***, ***j***, and ***k.***
d. Compute the dot product of two vectors; use dot products to compute the angle between two vectors, the component of one vector in the direction of another vector, or the orthogonal projection of one vector onto another.
e. Compute the cross product of two vectors; compute the scalar triple product of three vectors; compute the direction cosines and direction angles of a vector.
f. Use vector operations to find the equation of a plane; compute the shortest distance from a point to a plane.
g. Find the vector, parametric, or symmetric equations for a line in space.
h. Give the equation of a quadric surface in standard form, name the surface, and sketch its graph.
i. Sketch graphs of selected equations in cylindrical and spherical coordinates; re-express Cartesian equations in cylindrical or spherical form.

2. Students will be able to evaluate limits and continuity of vector-valued functions, differentiate vector-valued functions, and use differentiation and integration to solve practical problems involving vector-valued functions. Students will display proficiency by demonstrating the following competencies:

a. Find the domain of a vector-valued function; sketch the curve in space of selected vector-valued functions.
b. Compute the derivative of vector-valued functions; evaluate limits and integrals of vector-valued functions.
c. Compute the principal unit tangent and normal vectors to a curve; compute the arc length of selected curves.
d. Find the curvature vector of a curve; compute the curvature and radius of curvature of a curve at a point.
e. Find the velocity, acceleration, and speed associated with an equation of motion; compute the tangential and normal components of acceleration of an equation of motion.

3. Students will be able to evaluate limits and continuity of multivariate functions, and compute partial derivatives and multiple integrals of selected functions; additionally, students will be able to utilize the techniques of partial integration and multiple integration (together with appropriate technology) to solve practical problems and communicate results. Students will display proficiency by demonstrating the following competencies.

a. Find the domain of a function of several variables.
b. Describe and use level curves to sketch the graphs of selected functions of several variables.
c. Evaluate limits of functions of several variables; state the definition of continuity of a function of several variables at a point; determine whether a function of several variables is continuous at a point.
d. Compute first order and higher order partial derivatives of functions of several variables.
e. Compute the total differential of a function of several variables; use the total differential to approximate function values.
f. Apply the Chain Rule to differentiate functions of several variables.
g. Compute the directional derivative of a function in the direction of a vector; compute the gradient of a function.
h. Compute the vector normal to a surface at a given point; find the equation of the tangent plane to a surface at a given point.
i. Use partial differentiation to identify relative extrema of a function of several variables.
j. Use Lagrange multipliers to solve optimization problems.
k. Evaluate double integrals in rectangular or polar coordinates; use double integrals to compute volumes of solids and areas of regions.
l. Use double integrals to find surface areas of selected surfaces.
m. Compute the mass, the moments about the axes, and the center of mass of a plane lamina; compute the moments of inertia about the axes of a plane lamina.
n. Evaluate triple integrals in rectangular, cylindrical, or spherical coordinates; use triple integrals to compute volumes of solids.

4. Students will be able to carry out scalar and vector operations on vector-valued functions, and apply selected methods to evaluate line and surface integrals. Students will display proficiency by demonstrating the following competencies:

a. Compute the gradient, divergence, and curl of a vector-valued function; state the definition of a harmonic function and determine whether a function is harmonic.
b. Evaluate line integrals.
c. Use line integrals to compute the work done by a force moving an object along a curve.
d. State the definition of an exact differential and determine whether a differential is exact; state the definition of a conservative function and determine whether a function is conservative; state the definitions of open, closed, and simply connected and determine whether a region is open, closed, or simply connected; state the definition of independence of path and determine whether a line integral is independent of path.
e. Use Green's Theorem to evaluate line integrals.
f. Evaluate selected surface integrals.
g. Use the Divergence Theorem to evaluate surface integrals.
h. Use Stokes' Theorem to evaluate surface integrals.

OUTLINE OF INSTRUCTION:

I. Three-Dimensional Space; Vectors
   A. Rectangular Coordinates in 3-Space; Spheres; Cylindrical Surfaces
   B. Vectors
   C. Dot Product; Projections
   D. Cross Product
   E. Parametric Equations of Lines
   F. Planes in 3-Space
   G. Quadric Surfaces
   H. Cylindrical and Spherical Coordinates

II. Vector-valued functions
   A. Introduction to Vector-Valued Functions
   B. Calculus of Vector-Valued Functions
   C. Change of Parameter; Arc Length
   D. Unit Tangent, Normal, and Binormal Vectors
   E. Curvature
   F. Motion Along a Curve

III. Partial Derivatives
   A. Functions of Two or More Variables
   B. Limits and Continuity
   C. Partial Derivatives
   D. Differentiability, Differentials, and Local Linearity
   E. The Chain Rule
   F. Directional Derivatives and Gradients

G. Tangent Planes and Normal Vectors
H. Maxima and Minima of Functions of Two Variables
I. Lagrange Multipliers

IV. Multiple Integrals
A. Double Integrals
B. Double Integrals over Nonrectangular Regions
C. Double Integrals in Polar Coordinates
D. Parametric Surfaces; Surface Area
E. Triple Integrals
F. Centroid, Center of Gravity, Theorem of Pappus
G. Triple Integrals in Cylindrical and Spherical Coordinates
H. Change of Variables in Multiple Integrals, Jacobians

V. Topics in Vector Calculus
A. Vector Fields
B. Line Integrals
C. Independence of Path; Conservative Vector Fields
D. Green's Theorem
E. Surface Integrals
F. Applications of Surface Integrals; Flux
G. The Divergence Theorem
H. Stokes' Theorem

REQUIRED TEXTBOOK AND MATERIALS:

Anton, Howard, Irl Bivens, and Stephen Davis. Calculus Multivariable. 8th ed. Wiley, 2005.

TI-83/84 graphing calculator

STATEMENT FOR STUDENTS WITH DISABILITIES:

Students who require academic accommodations due to any physical, psychological, or learning disability are encouraged to request assistance from a disability services counselor within the first two weeks of class. Likewise, students who potentially require emergency medical attention due to any chronic health condition are encouraged to disclose this information to a disability services counselor within the first two weeks of class. Counselors can be contacted by calling 536-7207, ext. 1413 or by visiting the Student Development Office in the Phail Wynn Jr. Student Services Center, room 1309.